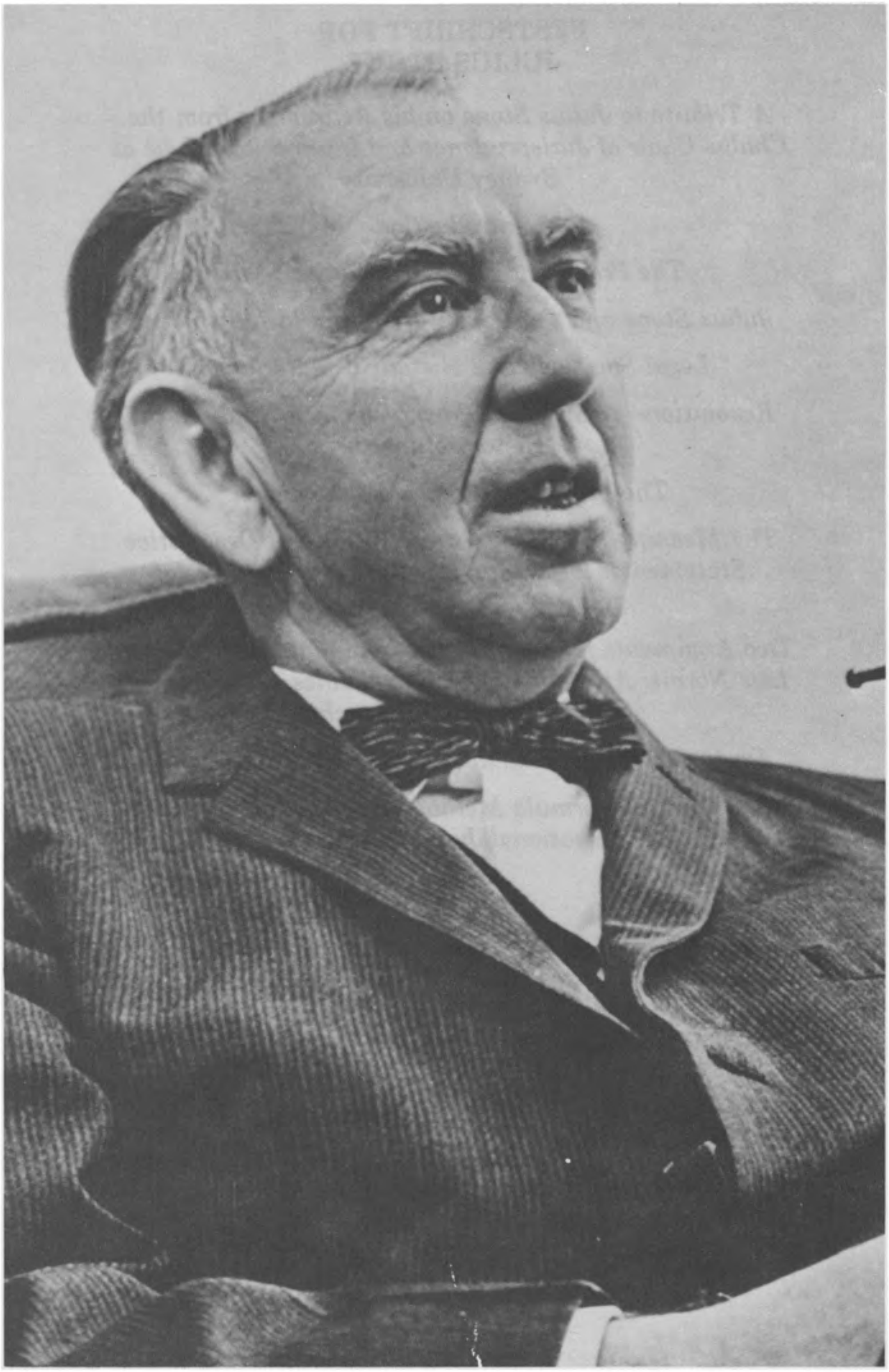
FESTSCHRIFT FOR JULIUS STONE

A Tribute to Julius Stone on his Retirement from the Challis Chair of Jurisprudence and International Law at Sydney University



THE COUNTER-FORMULA METHOD AND ITS APPLICATIONS IN INTERNATIONAL JUDICIAL REASONING

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I. INTRODUCTION

In the period when Julius Stone was the Head of the Department of International Law and Jurisprudence in the University of Sydney, pioneering work on the applications of modern logic to legal thought was done under his guidance. A notable result of this work was the devising of an efficient, logical decision-procedure of great practical utility to lawyers. In the present article I shall attempt to develop ideas originally propounded by Ilmar Tammelo and Ron Klinger,[1] who were members of that Department at that relevant time, and to apply the results of the development to samples of international judicial reasoning. In doing this I hope to join in the scholarly aspirations of Professor Stone, who not only has promoted work on legal logic but also has insisted on the need for overall enhancement of the intellectual quality of international legal thought. Thus he has pointed out that in view of the limited range of international judicial power and the corresponding necessity of restrained and careful reasoning, the contributions of international judges deficient in stringency of reasoning would lack the persuasive force so much required by the feebleness of the execution machinery of international judicial decisions.[2] An efficient logical decision-procedure, as the counter-formula method promises to be, would therefore be a much needed intellectual tool for international judges and other international lawyers.

The counter-formula method (C.F.M.) is a complete logical decision-procedure for propositional calculus (which calculus is the only one employed in this article) and for calculi having the same basic structure as the propositional calculus. This decision-procedure tests legal reasoning for its formal validity, solidity and

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1. *See* Tammelo & Klinger, *The Counter-Formula Method and Its Applications In Legal Logic* in DIMENSIONEN DES RECHTS: GEDACHTNISSCHRIFT FUR RENE MARCIC 349-60 (M. Fischer ed. 1974). The first published statement of the method is in I. TAMMELO, PRINCIPLES AND METHODS OF LEGAL LOGIC 31-36 (1971) (in Japanese). *See also* 1 I. TAMMELO & H. SCHREINER, GRUNDZUGE UND GRUNDVERFAHREN DER RECHTSLOGIK 39-44 (1974).

2. *See, e.g.*, Stone, *Non Liquet and the Function of Law in the International Community* 35 BRIT. Y.B. INT'L L. 124, 134-44 (1959). *See also* J. STONE, QUEST FOR SURVIVAL (1961).

compatibility as well as for its formal invalidity, insolidity and incompatibility. The C.F.M. offers more than the indirect deductive proof (to which it is similar) because it is capable of leading to the logical decision in every instance of its application, which the indirect proof (like the direct proof and the conditional proof) cannot do.

In contrast to the commonly used logical decision-procedures (*e.g.*, the tabular methods and the normal-forms methods), the C.F.M. can be relatively easily handled in those cases in which a considerable number of variables are involved in their logical expressions. The application of this method does not produce excessively long formulae (as, for example, the normal-forms methods do) but produces formulae which become progressively shorter as the execution of the method proceeds. Therefore, clerical errors can be easily avoided or discovered in the C.F.M. derivation schemata. Moreover, the method can always be executed on normal writing or printing paper and does not require sheets of excessive size, as diagrammatic decision-procedures (for example, the "truth-tree" method) would require in some instances of its application to cases of great legal significance.

The Polish notation is here employed with slight modifications introduced by Ilmar Tammelo. In place of **N** for the negator, a bar above the negated sign is here used and for the injunctor, **D** has been adopted. This notation proves to be most expedient in the execution of the C.F.M. (and of any other decision-procedure of practical significance) since it provides the shortest possible and most easily readable and surveyable formulae.

As a useful novel term, "dyslogy" is employed here for characterizing those formulae whose ultimate value constellation is "minus" (or "false" in indicative logic). Dyslogy—the negation of tautology—corresponds to what is commonly called "self-contradiction." For characterising those formulae which are neither tautologous nor dyslogous, the term "amphilogy" is apt; it corresponds to what is commonly called "logical contingency." As terms with special technical meaning "solid" and "insolid" are here adopted, the former to characterize a conclusion that follows from a non-dyslogous derivation basis (*i.e.* premise or premises) and the latter to characterize a conclusion that follows from a dyslogous derivation basis. Further, "compatibility" is employed here to characterize a conclusion which is consistent with its derivation basis and "incompatibility" to characterize a conclusion which is inconsistent with its derivation basis.

The terms employed here for dyadic operators and for the corresponding functions are adopted from Paul Lorenzen, whose pertinent terminology is not only uniform and elegant but is also fertile in that it supplies useful terminological derivations. Accordingly, the **C**-operator (if . . . then . . .) is called "subjunctor," the **A**-operator (. . . or . . .) "adjunctor," the **K**-operator (. . . and . . .) "conjunctor," the **E**-operator (if and only if . . . then . . .) "bijunctor," and the **D**-operator (only if . . . then . . .) "injunctor."[3] The corresponding functions are called "subjunction," "adjunction," etc. For the expression of the negative junctors, the prefix "contra" is used here (hence, for example, "contra-adjunctor" or "contrainjunctor").

II. THE RULES OF THE COUNTER-FORMULA METHOD

The term "counter-formula" plays a central role in the exposition of the counter-formula method. It means a formula which differs from another formula only by a negative (a bar) on the top of its first sign (for example, $\bar{R}$ is a counter-formula of R and vice versa; **A**PQ is a counter-formula of $\bar{\mathbf{A}}$PQ and vice versa).

The procedure of the counter-formula method is carried out according to the transcription and elimination rules. The former are to produce formulae containing only adjunctors and conjunctors, whereas the latter are to effect the breaking of longer formulae into shorter ones. The objective of the C.F.M. is reached either when, after the application of all the relevant rules, a counter-formula for any formula appearing in the derivation basis or among the derivations is produced, or when all the relevant rules have been exhausted and this has not been achieved. In the first case it is proved that the conclusion validly follows from the derivation basis; in the second case it is proved that the claimed conclusion is logically invalid.

In the examples that are attached to the statement of the transcription rules, the variables X, Y and Z are used to stand for any well-formed formula—either simple or complex.

A. The Transcription Rules

1. Wherever there is a formula with more than one negator,

3. The term "injunctor" has not been used so far in the works of Paul Lorenzen. He has suggested it in reply to a letter seeking his terminological advice.

cancel any two of them and write the formula without a negator or with one negator respectively (Double Negation—D.N.). For ple, from $\bar{\bar{x}}$ follows x; from $\bar{\bar{\bar{\mathbf{A}}}}xy$ follows $\bar{\mathbf{A}}xy$.

2. Wherever an adjunctor governs exactly the same components, write one component only (Autology—Aut.). For example, from $\mathbf{A}xx$ follows x; from $\mathbf{AA}xy\mathbf{A}xy$ follows $\mathbf{A}xy$.

3. In place of $\mathbf{C}xy$ write $\mathbf{A}\bar{x}y$ (Duality for Subjunction—S.Dual.).

4. In place of $\mathbf{D}xy$ write $\mathbf{A}x\bar{y}$ (Duality for Injunction—I.Dual.).

5. In place of $\mathbf{E}xy$ write $\mathbf{KA}\bar{x}y\mathbf{A}x\bar{y}$ (Dissection for Bijunction—B.Diss.).

6. In place of $\bar{\mathbf{C}}xy$ write $\mathbf{K}x\bar{y}$ (Duality for Contrasubjunction—CS.Dual.).

7. In place of $\bar{\mathbf{D}}xy$ write $\mathbf{K}\bar{x}y$ (Duality for Contrainjunction—CI.Dual.).

8. In place of $\bar{\mathbf{A}}xy$ write $\mathbf{K}\bar{x}\bar{y}$ (Duality for Contraadjunction—CA.Dual.).

9. In place of $\bar{\mathbf{K}}xy$ write $\mathbf{A}\bar{x}\bar{y}$ (Duality for Contraconjunction—CC.Dual.).

10. In place of $\bar{\mathbf{E}}xy$ write $\mathbf{KA}xy\mathbf{A}\bar{x}\bar{y}$ (Dissection for Contrabijunction—CB.Diss.).

B. The Elimination Rules

1. Wherever a conjunction appears separately, write its conjuncts as separate entries; wherever a conjunction appears in an adjunction, write its conjuncts separately in an adjunction. (Conjunction Elimination—C.El.) For example, from $\mathbf{K}xy$ follows x and y; from $\mathbf{A}x\mathbf{K}yz$ follows $\mathbf{A}xy$ and $\mathbf{A}xz$.

2. Wherever an adjunct has a counter-formula separately, write the other adjunct as a separate entry; wherever an adjunct has a counter-formula in an adjunction, write the other adjuncts in an adjunction. (Adjunction Elimination—A.El.) For example, from $\mathbf{A}xy$ and $\bar{y}$ follows x; from $\mathbf{A}xy$ and $\mathbf{A}x\mathbf{A}\bar{y}z$ follows $\mathbf{A}xz$.

According to the commutation and the association laws of the propositional calculus, the position of the adjunctors and the conjunctors as well as of the components they govern can be rearranged provided that the principles determining the well-formed formulae are observed. The principle under which this rearrangement can be effected is here called "Position Rearrangement" and for this term the abbreviation "P.R." is used. For example, $\mathbf{KA}xyx$ can be rearranged as **K**x**A**xy or as **K**x**A**yx, from which under the rule of Conjunction Elimination follows x and **A**xy or x and **A**yx respectively.

III. THE PROCEDURE OF THE COUNTER-FORMULA METHOD

For the execution of the counter-formula method, a derivation schema is established by writing first the derivation basis (consisting of the premise or premises of the argument) and under it the formulae, one after another, which follow from the application of the C.F.M. to the formulae belonging to the derivation basis or to the formulae derived therefrom. All these entries are written in the left-hand column; the information about them is provided in the right-hand column. Accordingly, the derivation schema contains a derivation column and an information column. After each derived entry (occurring in the derivation column) the number or numbers on which the step rests together with the abbreviated name of the rule employed for its justification are written (in the information column).

A. Proof of Validity

For ascertaining by means of the C.F.M. whether the conclusion of an argument is valid, proceed as follows:

> Write down the premise or premises one after the other and number them. As the entry immediately following the last premise, write the counter-formula of the conclusion and provide it with the number consecutive to the number of the last premise. Below this entry write all entries derived from the application of the C.F.M. to any premise or to the counter-formula of the conclusion. In the information column, write the conclusion of the argument on the same line as the last premise. Below the conclusion, provide the requisite informations for the entries in the derivation column. As the first information, write "C.F.C." (for "counter-formula of the conclusion").

If after the application of the relevant rules a counter-formula for any formula in the derivation column is attained, then the conclusion is valid. If after the exhaustion of all relevant rules this does not happen, then the conclusion is invalid. For example:

1.	**CK**P$\bar{\mathrm{Q}}\bar{\mathrm{R}}$	
2.	**CK**SVP	
3.	**C**$\bar{\mathrm{W}}$Q	
4.	**KK**S$\bar{\mathrm{W}}$V	∴ $\bar{\mathrm{R}}$
5.	R	C.F.C.

6.	$\mathbf{A}\overline{\mathbf{K}}\mathrm{p}\overline{\mathrm{q}}\overline{\mathrm{r}}$	1, S. Dual.
7.	$\mathbf{AA}\overline{\mathrm{p}}\mathrm{q}\overline{\mathrm{r}}$	6, CC.Dual., D.N.
8.	$\mathbf{A}\overline{\mathrm{p}}\mathrm{q}$	5, 7, A.El.
9.	$\mathbf{A}\overline{\mathbf{K}}\mathrm{svp}$	2, S.Dual.
10.	$\mathbf{AA}\overline{\mathrm{s}}\overline{\mathrm{v}}\mathrm{p}$	9, CC.Dual.
11.	$\mathbf{A}\mathrm{w}\overline{\mathrm{q}}$	3, S.Dual., D.N.
12.	$\mathbf{A}\overline{\mathrm{p}}\mathrm{w}$	8, 11 A.El.
13.	$\mathbf{AA}\overline{\mathrm{s}}\overline{\mathrm{v}}\mathrm{w}$	10, 12, A.El.
14.	$\mathrm{s}, \overline{\mathrm{w}}, \mathrm{v}$	4, C.El.
15.	$\mathbf{A}\overline{\mathrm{v}}\mathrm{w}$	13, 14 A.El.
16.	w	14, 15 A.El.

Step (16) produced a counter-formula for a formula appearing under (14). The above stated conclusion is therefore valid. Note that under (14) the entries were written one after the other separated by commas (and not one under the other). This was in order to save space.

B. *Proof of Insolidity*

To ascertain whether the conclusion of an argument is insolid, proceed as follows:

> Treat the derivation basis of the argument by the C.F.M. without positing the counter-formula of the conclusion as an entry.

If in the course of the procedure a counter-formula appears for any formula in the derivation column, then the conclusion is insolid. If after the exhaustion of all relevant rules this does not happen, then the conclusion is solid. For example:

1.	$\mathbf{C}\mathrm{p}\mathbf{A}\mathrm{qr}$	
2.	$\overline{\mathbf{A}}\mathbf{A}\mathrm{s}\overline{\mathrm{q}}\overline{\mathrm{s}}$	
3.	$\overline{\mathrm{v}}$	/∴ Cpr.
4.	$\mathbf{K}\overline{\mathbf{A}}\mathrm{s}\overline{\mathrm{q}}\mathrm{s}$	2, CA.Dual.
5.	$\overline{\mathbf{A}}\mathrm{s}\overline{\mathrm{q}}$	4, C.El.
6.	$\mathbf{K}\overline{\mathrm{s}}\mathrm{q}$	5, CA. Dual., D.N.
7.	$\overline{\mathrm{s}}$	6, C.El.
8.	s	4, C.El.

Step (8) produced a counter-formula for the formula appearing under (7). This demonstrates that the premises of the argument form a dyslogy, from which any conclusion can be derived (under the *Ex Falso Quodlibet* theorem). The above stated conclusion is therefore insolid. Note that only the treatment of the second premise according to the C.F.M. rules produced this result; it was thus not necessary to treat the other premises.

C. Proof of Incompatibility

For ascertaining whether the conclusion of an argument is incompatible with its derivation basis, proceed as follows:

> Write down the conclusion of a solid argument immediately below the last premise and indicate this step by "I.C." ("insertion of the conclusion") in the information column.

If in the course of the procedure a counter-formula appears for any other formula in the derivation column, then the conclusion is incompatible with the derivation basis. In this case the whole argument forms a dyslogy and it is hence self-contradictory. If a counter-formula for any formula in the derivation column cannot be attained after the exhaustion of all relevant rules, then the conclusion is compatible with its premises.[4] For example:

1.	$CKP\bar{Q}\bar{R}$	
2.	$CKSVP$	
3.	$AW\bar{Q}$	
4.	$KS\bar{W}$	
5.	V	/∴ R
6.	R	I.C.
7.	$A\overline{K}P\bar{Q}\bar{R}$	1, S.Dual.
8.	$AA\bar{P}Q\bar{R}$	7, CC.Dual., D.N.
9.	$A\bar{P}Q$	6, 8, A.E1.
10.	$A\overline{K}SVP$	2, S.Dual.
11.	$AA\overline{SV}P$	10, CC.Dual.
12.	$A\bar{S}P$	5, 11, A.El.
13.	$AQ\bar{S}$	9, 12 A.El.
14.	S	4, C.El.
15.	Q	13, 14 A.El.
16.	W	3, 15 A.El.
17.	$\bar{W}$	4, C.El.

Step (17) produced a counter-formula for the formula appearing under (16). Hence the conclusion is incompatible with its derivation basis.

D. Proof of Invalidity

1.	APQ	
2.	$\bar{R}$	
3.	$K\overline{SV}$	
4.	$C\bar{S}KWU$	
5.	CUA	/∴ $C\bar{R}K\bar{S}\bar{A}$

4. The finding that an argument is incompatible imports its reinforced condemnation. It is otherwise scarcely significant, because an incompatible argument is in any event invalid.

6.	$\overline{\mathbf{C}}\overline{R}\mathbf{K}\overline{S}\overline{A}$	C.F.C.
7.	$\mathbf{K}\overline{R}\overline{\mathbf{K}}\overline{S}\overline{A}$	6, CS.Dual.
8.	$\mathbf{A}SA$	7, C.El., CC. Dual., D.N.
9.	$\overline{S}$	3, C.El.
10.	A	8, 9 A.El.
11.	$\overline{V}$	3, C.El.
12.	$\mathbf{A}S\mathbf{K}WU$	4, S.Dual., D.N.
13.	$\mathbf{K}WU$	9, 12, A.El.
14.	W	13, C.El.
15.	U	13, C.El.
16.	$\mathbf{A}SW$	12, C.El.
17.	$\mathbf{A}SU$	12, C.El.
18.	$\mathbf{A}\overline{U}A$	5, S.Dual.
19.	W	9, 16, A.El.
20.	U	9, 17, A.El.
21.	A	15, 18, A.El.

This exhausts the application of all relevant rules. Yet for any formula no counter-formula has appeared in the course of the procedure. Note that Steps (19) and (20) could have been dispensed with since they could produce only what Steps (14) and (15) had already produced. Hence the conclusion here is invalid.

E. Proof of Solidity

1.	$\mathbf{DK}PQR$	
2.	$\mathbf{A}\overline{V}\mathbf{K}\overline{R}P$	
3.	$\mathbf{AK}WAR$	
4.	$\mathbf{K}\overline{S}V$	∴ $\mathbf{K}W\overline{R}$
5.	$\mathbf{AK}PQ\overline{R}$	1, I.Dual.
6.	$\mathbf{A}P\overline{R}$	5, C.El.
7.	$\mathbf{A}Q\overline{R}$	5, C.El.
8.	$\mathbf{A}\overline{V}\overline{R}$	2, C.El.
9.	$\mathbf{A}\overline{V}P$	2, C.El.
10.	$\mathbf{A}WR$	3, C.El.
11.	$\mathbf{A}AR$	3, C.El.
12.	$\mathbf{A}PA$	6, 11, A.El.
13.	$\mathbf{A}QA$	7, 11, A.El.
14.	$\mathbf{A}\overline{V}A$	8, 11, A.El.
15.	$\mathbf{A}\overline{V}W$	8, 10, A.El.
16.	$\mathbf{A}PW$	6, 10, A.El.
17.	$\mathbf{A}QW$	7, 10, A.El.
18.	$\overline{S}$	4, C.El.
19.	V	4, C.El.
20.	$\overline{R}$	8, 19, A.El.
21.	A	11, 20, A.El.
22.	W	15, 19, A.El.
23.	P	9, 19, A.El.

Here all relevant rules except those which would have produced exactly the same results that were already achieved have been exhausted. Yet no counter-formula appeared for any formula in the course of the procedure. Therefore, the conclusion is solid. The above conclusion can also be shown to be valid.

F. Proof of Compatibility

1.	$\overline{\mathbf{A}\overline{\mathbf{E}}PQ}\ \overline{\mathbf{E}}QR$	
2.	$\mathbf{C}RV$	$/\therefore\ \mathbf{A}\bar{Q}V$
3.	$\mathbf{A}\bar{Q}V$	I.C.
4.	$\mathbf{K}\mathbf{E}PQ\mathbf{E}QR$	1, C.A.Dual., D.N.
5.	$\mathbf{E}PQ$	4, C.El.
6.	$\mathbf{E}QR$	4, C.El.
7.	$\mathbf{A}\bar{P}Q$	5, B.Diss., C.El.
8.	$\mathbf{A}P\bar{Q}$	5, B.Diss., C.El.
9.	$\mathbf{A}\bar{Q}R$	6, B.Diss., C.El.
10.	$\mathbf{A}Q\bar{R}$	6, B.Diss., C.El.
11.	$\mathbf{A}\bar{R}V$	2, S.Dual.
12.	$\mathbf{A}V\bar{P}$	3, 7 A.El.
13.	$\mathbf{A}VR$	3, 10, A.El.
14.	$\mathbf{A}P\bar{P}$	7, 8 A.El.
15.	$\mathbf{A}Q\bar{Q}$	7, 8 A.El.
16.	$\mathbf{A}\bar{P}R$	7, 9 A.El.
17.	$\mathbf{A}P\bar{R}$	8, 10, A.El.
18.	$\mathbf{A}\bar{Q}V$	8, 12, A.El.
19.	$\mathbf{A}R\overline{R}$	9, 10, A.El.

Further application of the relevant rules here would produce only formulae already attained by previous steps. Thus the application of all relevant rules can be considered as exhausted. Since no counter-formula appeared for any formula in the course of the procedure, the conclusion is compatible with its derivation basis.

Note that the application of the Adjunction Elimination rule produced tautologous formulae with Steps (14), (15) and (19). Since such formulae lead only to repetitions of otherwise produced formulae in the C.F.M., their writing down is dispensable.

IV. APPLICATIONS OF THE COUNTER-FORMULA METHOD IN INTERNATIONAL LEGAL REASONING

In the following analysis of the logical aspect of some international legal cases, the structurization of the arguments—which is always a matter of construction or interpretation of relevant expressions rather than a matter of their logical treatment—is based on the writer's understanding of the court's statements of reasons for their decisions. A different understanding of these statements may lead to a different structurization, and correspondingly to a different formalization, of the arguments. Whatever structurization may ultimately prove to be tenable, the present understanding of the statements of reasons in question is sufficient for the illustrative purposes of the application of the C.F.M. in the field of international legal reasoning.

A. *The Lotus Case*

[1927] P.C.I.J., ser. A, No. 10.

Facts: This case grew out of a collision which occurred on 2 August 1926 between the French mail steamer *Lotus* proceeding to Constantinople and the Turkish collier *Boz-Kourt*. The *Boz-Kourt* was cut in two, sank and several Turkish nationals perished. After having tried to save persons, the *Lotus* continued its course to Constantinople. A few days later, the officer of watch on board the *Lotus* was placed under arrest. The French government contended that a principle of international law prohibited the Turkish government from prosecuting its nationals.

The Court: Though it is true that in all systems of law the principle of the territorial character of criminal law is fundamental, it is equally true that all or nearly all these systems of law extend their action to offenses committed outside the territory of the State which adopts them, and they do so in ways which vary from State to State. The territoriality of criminal law, therefore, is not an absolute principle of international law and by no means coincides with territorial sovereignty.

1. The Structure of the Argument

In all systems of law, the principle of the territorial character of criminal law is fundamental. All or nearly all systems of law extend their action to offenses committed outside the territory of the State which adopts them. They extend their action to offenses committed outside the territory of the State in ways which vary from State to State. If all or nearly all systems of law extend their action to offenses committed outside the territory of the State which adopts them and all systems of law extend their action to offences committed outside the territory of the State in ways which vary from State to State, then the principle of the territorial character of criminal law is not absolute. If the principle of the territorial character of international law is not absolute, then it does not coincide with territorial sovereignty. Therefore, the principle of territorial character of criminal law is not absolute and it does not coincide with territorial sovereignty.

2. Glossary

In the following glossary, small size lower-case letters are employed, in contrast to the small size upper-case letters employed in the above examples of the C.F.M. proofs. The letters employed in examples signify any propositions whatsoever, whereas the letters employed hereinafter signify the instances of given propositions.

p: In all systems of law the principle of the territorial character of criminal law is fundamental.

q: All systems of law extend their action to offenses committed outside the territory of the State which adopts them.

r: Nearly all systems of law extend their actions to offenses committed outside the territory of the State which adopts them.

s: All systems of law extend their action to offenses committed outside the territory of the State in ways which vary from State to State.

a: The principle of the territorial character of criminal law is absolute.

c: The principle of the territorial character of criminal law coincides with territorial sovereignty.

3. Proof of Validity

1.	p	
2.	$\mathbf{KA}qrs$	
3.	$\mathbf{CKA}qrs\bar{a}$	
4.	$\mathbf{C}\bar{a}\bar{c}$	/∴ $\underline{\mathbf{K}\bar{a}\bar{c}}$
5.	$\overline{\mathbf{K}}\bar{a}\bar{c}$	C.F.C.
6.	$\mathbf{A}ac$	5, CC.Dual., D.N.
7.	$\mathbf{A}a\bar{c}$	4, S.Dual., D.N.
8.	a	6, 7 A.El., Aut.
9.	$\mathbf{A}\overline{\mathbf{K}}\mathbf{A}qrs\bar{a}$	3, S.Dual.
10.	$\overline{\mathbf{K}}\mathbf{A}qrs$	8, 9 A.El.
11.	$\mathbf{A}\overline{\mathbf{A}}qr\bar{s}$	10, CC.Dual.
12.	$\mathbf{AK}\bar{q}\bar{r}\bar{s}$	11, CA.Dual.
13.	s	2, C.El.
14.	$\mathbf{K}\bar{q}\bar{r}$	12, 13, A.El.
15.	$\bar{q}$	14, C.El.

16.	$\bar{r}$	14, C.El.
17.	Aqr	2, C.El.
18.	q	16, 17, A.El.

Step (18) produced a counter-formula for the formula appearing under (15). Therefore, the conclusion of the above argument is valid.

4. Proof of Solidity

In order to execute the proof of solidity of the above conclusion, the premises from which it was derived need not be written down again. The procedure required for this proof starts with Step (5).

5.	Aqr	2, C.El.
6.	s	2, C.El.
7.	$A\bar{K}Aqrs\bar{a}$	3, S.Dual.
8.	$AA\bar{A}qr\bar{s}\bar{a}$	7, CC.Dual.
9.	$AAK\bar{q}\bar{r}\bar{s}\bar{a}$	8, CA.Dual.
10.	$AA\bar{q}\bar{s}\bar{a}$	9, C.El.
11.	$AA\bar{r}\bar{s}\bar{a}$	9, C.El.
12.	$Aa\bar{c}$	4, S.Dual., D.N.
13.	$A\bar{q}\bar{a}$	6, 10, A.El.
14.	$A\bar{r}\bar{a}$	6, 11, A.El.
15.	$A\bar{c}\bar{q}$	12, 13, A.El.
16.	$A\bar{c}\bar{r}$	12, 14, A.El.

The application of all relevant rules has now been exhausted; yet no counter-formula appeared for any formula in the course of the procedure. The conclusion of the above argument is therefore solid.

5. Proof of Compatibility

In order to execute the proof of compatibility of this conclusion with its derivation basis, the above Proof of Solidity can be used. The procedure is continued by inserting the conclusion in the derivation column by Step (17).

17.	Kac	I.C.
18.	$\bar{a}$	17, C.El.
19.	$\bar{c}$	17, C.El.

It is not possible to proceed further under the relevant rules. Since no counter-formula appeared for any formula in the course of procedure, the conclusion of the above argument is compatible with its derivation basis.

B. Apostolidis v. The Turkish Government

8 Trib. Arb. Mixtes 373 (1928).

Facts: A claim was made to the Franco-Turkish Mixed Arbitral Tribunal by the plaintiff, who was a French citizen resident in Athens, for the restitution of certain properties inherited in Turkey from his father. The Turkish government, which had retained the properties, challenged the jurisdiction of the Tribunal alleging that the plaintiff had retained Turkish nationality under Article 5 of the Turkish law of January 19, 1869, providing that naturalization of a Turkish national without a previous authorization of the Turkish government is considered null and void.

The Court: The defendant has invoked Article 5 of the Turkish law of January 19, 1869, providing that the naturalization of an Ottoman subject without a previous authorization of the Imperial Government shall be considered null and void. On the basis of this Article, the defendant maintains that Athenodore, who had not obtained the required authorization, had retained his Turkish nationality without acquiring French nationality, and consequently the Tribunal lacked jurisdiction to decide the plaintiff's claim. According to the principles of public international law, the effects of naturalization must be recognized not only by the authorities of the state which has granted this naturalization but equally by the judicial and administrative authorities of all other states. In the exceptional case where the laws of a state require previous governmental authorization for the naturalization of its nationals abroad, it is only the authorities of that state who are bound to regard an unauthorized naturalization as invalid. It follows in the present case that, although the Turkish administrative and judicial authorities were entitled to refuse to recognize the effects of the naturalization, all *other* judicial authorities are bound to recognize the validity of the change of nationality and to recognise the claimants as French nationals. Therefore, the Tribunal declares that it has jurisdiction.

1. THE STRUCTURE OF THE ARGUMENT

If according to Article 5 of the Turkish law of January 19, 1869, previous authorization for naturalization is not given, then the naturalization is considered null and void. If this authorization is not given then the plaintiff did not acquire French nationality according to Turkish law and the Mixed Arbitral Tribunal lacked jurisdiction to decide the plaintiff's claim. The effects of naturalization must be recognized by the authorities of the state which has granted this naturalization, and these effects must be recognized equally by the judicial and administrative authorities of all other states. If the laws of a state require previous governmental authorization for the naturalization of its nationals abroad, then the authorities of that state are bound to regard an unauthorized naturalization as invalid. The Turkish administrative and judicial authorities were entitled to refuse to recognize the effects of the naturalization, and all judicial and administrative authorities of all other states must recognize the effects of a naturalization, and all other judicial authorities are bound to recognize the validity of the change of nationality. Therefore, the Tribunal has the jurisdiction to decide the plaintiff's claim.

2. Glossary

a: A previous authorization according to Article 5 of the Turkish law of January 19, 1869, is given.

v: The naturalization is considered null and void.

q: The plaintiff acquired French nationality according to French law.

r: The effects of the naturalization must be recognized by the authorities of the state which has granted this naturalization.

n: The Turkish administrative and judicial authorities were entitled to refuse to recognize the effects of the naturalization.

e: The effects of the naturalization must be recognized by the judicial and administrative authorities of all other states.

i: The law of a state requires previous governmental authorization.

o: The authorities of that state are bound to regard an unauthorized naturalization as invalid.

w: All other judicial authorities are bound to recognize the claimants as French nationals.

u: The Tribunal has jurisdiction to decide the plaintiff's claim.

3. Proof of Validity

1. $\mathbf{C}\bar{a}v$
2. $\mathbf{C}\bar{a}\mathbf{K}\bar{q}\bar{u}$
3. $\mathbf{K}re$
4. $\mathbf{C}io$
5. $\mathbf{K}n\mathbf{K}ew$ $\quad / \therefore \underline{u}$

6.	$\bar{u}$	C.F.C.
7.	**Aav**	1, S.Dual., D.N.
8.	**AaK**$\bar{q}\bar{u}$	2, S.Dual., D.N.
9.	**Aa**$\bar{q}$	8, K.El.
10.	**Aa**$\bar{u}$	8, K.El.
11.	r	3, K.El.
12.	e	3, K.El.
13.	**A**$\bar{I}$o	4, S.Dual.
14.	n	5, K.El.
15.	e	5, K.El.
16.	w	5, K.El.

This exhausts the application of all relevant rules. Yet for any formula no counter-formula has appeared in the course of the procedure. Hence, the conclusion of the above argument is invalid. This result is due above all to the fact that the court's reasoning in the present case is enthymematic, that is, it suppresses some premises necessary for supporting a valid and solid conclusion. Further, it appears that predicational rather than propositional calculus would have been better suited to formalize the argument (which formalization is beyond the present scope).

C. *Mortensen v. Peters*

8 Sess. Cas. 93 (1906).

Facts: This case was a test case of the extent of British jurisdiction to prohibit trawl-fishing in the Moray Firth, irrespective of the nationality of the offender of the vessel. The appellant was a Dane and was charged with having contravened the statutes and bylaws. Those statutes, alleged to have been contravened, being British municipal legislation, only conferred jurisdiction over (a) British subjects and (b) foreign subjects within British territory.

Counsel for the Appellant: The statutes creating offenses must be presumed to apply only (1) to British subjects and (2) to foreign subjects in British territory. Short of express enactment their application should not be further extended. The appellant is admittedly not a British subject, which excludes (1); the *locus delicti,* being in the sea beyond the three-mile limit, was not within British territory. Consequently, the appellant was not included in the prohibition of the statute.

1. The Structure of the Argument

Statutes creating offenses must be presumed to apply only to

British subjects and they must be presumed to apply only to foreign subjects in British territory; and if there is no express enactment, then their application should not be further extended. If the appellant is not a British subject and statutes creating offenses must be presumed to apply only to British subjects, then the appellant was not included in the prohibition of the statute. If the *locus delicti* is not within British territory and statutes creating offenses must be presumed to apply only to foreign subjects in British territory, then the appellant was not included in the prohibition of the statute. The appellant is not a British subject and the *locus delicti* is not within British territory. Therefore, the appellant is not included in the prohibition of the statute.

2. Glossary

p: Statutes creating offenses must be presumed to apply to British subjects.

q: Statutes creating offenses must be presumed to apply to foreign subjects in British territory.

r: There is express enactment.

a: The application should be extended.

s: The appellant is a British subject.

v: The *locus delicti* is within British territory.

i: The appellant is included in the prohibition of the statute.

3. Proof of Validity

1.	$\mathbf{KK}pq\mathbf{C}\bar{r}\bar{a}$	
2.	$\mathbf{CK}\bar{s}p\bar{i}$	
3.	$\mathbf{CK}\bar{v}q\bar{i}$	
4.	$\mathbf{K}\bar{s}\bar{v}$	$/\therefore \bar{i}$
5.	i	C.F.C.
6.	p	1, C.El.
7.	$\mathbf{A}\overline{\mathbf{K}}spi$	2, S.Dual.
8.	$\mathbf{AA}s\bar{p}\bar{i}$	7, CC.Dual., D.N.
9.	$\bar{s}$	4, C.El.
10.	$\mathbf{A}\bar{p}\bar{i}$	8, 9, A.El.
11.	$\bar{p}$	5, 10, A.El.

Step (11) produces a counter-formula for the formula appearing under (6). Hence the conclusion of the above argument is valid. This conclusion can also be shown to be solid and compatible with its premises.